Magnetohydrodynamic Generators with Nonequilibrium Ionization

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Introduction

THE earliest serious study of the possibility of utilizing nonequilibrium ionization in MHD generators was carried out by Karlowitz and Halasz at the Westinghouse Research Laboratories[1] between 1938 and 1947. It was originally oriented to the development of a generator that could use combustion products at moderate temperatures. The study was terminated when it was realized that recombination rates in molecular gases were too high to permit useful electron densities to be developed. It was concluded, however, that nonequilibrium ionization might be feasible in the atomic working gases that can be used with a nuclear reactor. This work was reviewed by Karlowitz in 1962.[2]

With recent developments in nuclear reactor technology, there is now renewed interest in nuclear-powered MHD generators. The temperature range, which is likely to be accessible with reactors and also desirable thermodynamically, is from about 1500° to 2000°K. It seems likely that cycles with over-all efficiencies between 0.50 and 0.60 can be evolved with such maximum temperatures. The requirements of the reactor and the generator together dictate that the pressure in the generator be between 1 and perhaps 10 atm, if the working fluid is argon and the magnetic field is limited to, say, 20,000 gauss. With the development of large superconducting magnets, it may be possible to raise these pressures by a factor of 5. It is likely that the inert gas will be seeded with a fraction of a percent of cesium.

Since 1960, considerable effort has been devoted to the study of such plasmas. It began with small-scale studies of the behavior of the plasma, and has only recently advanced to the study of complete generators, the point at which Karlowitz and Halasz left off. The purpose of this article is to review this work and to summarize the current understanding of nonequilibrium ionization, as applied to MHD power generation. This is an important qualification, since the literature of nonequilibrium ionization taken in the broad sense is vast indeed. It includes the literature of gas-discharge physics and low-density plasma physics. As we shall show, however, the nonequilibrium plasmas of interest in MHD power generation are much simpler than those of lower density. The essential point is that the behavior of the high-density plasma is controlled by volumetric phenomena, not by surface phenomena. This distinguishes it particularly from the glow-discharge plasmas so thoroughly studied in the past.

There are a number of ways of producing a homogeneous plasma with nonequilibrium ionization. To name a few: there are the methods of photoionization, electron-beam ionization, radio-frequency excitation, radioactive decay, and, finally, nonequilibrium ionization due to (d.c.) Joule heating. It is the last of these methods which will occupy most of the present discussion. Many of the conclusions to be drawn, however, can be extended to the other methods. This follows from the fact, to be demonstrated below, that the general question of the feasibility of producing nonequilibrium ionization can be separated into two fairly distinct parts. There is first the question of the behavior of the nonequilibrium plasma, given that a certain input power is provided to maintain the electrons out of equilibrium with the gas. The question can be posed as follows: 1) What is the connection between the local electrical conductivity of the gas and the local (dissipative) power density? We do not have a detailed theoretical answer to this question, which is very complex, but we do have a completely satisfactory engineering answer, which applies to most of the methods of ionization mentioned previously.

The second question is as follows: 2) Can the local (dissipative) power density and gas composition be so chosen in a practical MHD generator configuration as to satisfy the conditions required to produce nonequilibrium ionization? This second question is in many ways more complex and difficult than the first, and the problem is different for each of the ionization schemes mentioned previously. We are certainly farther from having a satisfactory answer, although recent progress in the application of the scheme utilizing the natural d.c. Joule heating of the electrons is encouraging.

The following discussion will be divided into two parts. The first, entitled "Nonequilibrium Plasma Behavior," will deal with question 1; the second, entitled "Generator Characteristics," will deal with question 2.

Most of the discussion will apply to alkali-metal seeded noble gases, since these have received the greatest attention. Some work has been done on wet alkali-metal vapors, however, and these results will be mentioned briefly.

A serious attempt has been made to ensure that the discussion is correct both historically and technically. The

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Received September 15, 1964; revision received November 4, 1964.

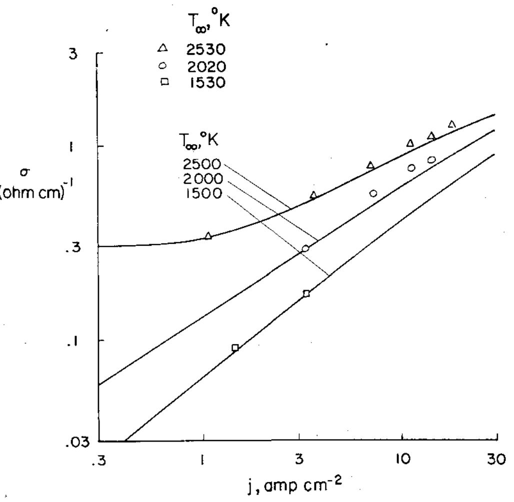

Fig. 1 Comparison of two-temperature conduction theory with measurements taken in flowing potassium-seeded argon. The theory is given for an effective $\delta = 24$, rather than 3.5 as originally reported, and a total cross section of 7.2×10^{-16} cm^2 (see Ref. 3).

author will be most grateful for the receipt of any corrections or additions.

I. Nonequilibrium Plasma Behavior

As noted previously, the nonequilibrium plasmas under discussion here are essentially different from those found in glow discharges even though they share the characteristic that the electron temperature is elevated above the gas temperature. One essential difference is that, in the dense plasmas of interest for MHD generators, the properties of the plasma are determined by a balance between electron energy gain and loss, which is nearly local. One could argue that this must be the case if the plasma is to be of use for power generation, because the MHD generator must be insensitive to surface effects in order to be efficient. In fact, we shall find that the plasma is locally determinate except for the effects of radiant energy loss, and this effect should be small in generators of useful size.

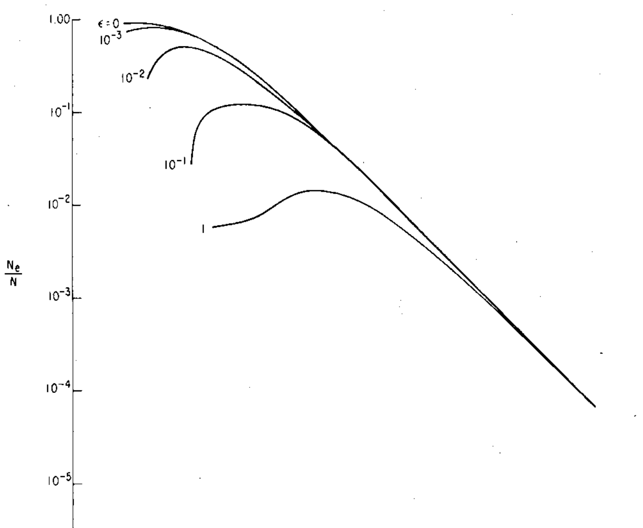

Fig. 2 The degree of ionization of cesium-seeded helium as a function of the cesium concentration, N(cm^{-3}) and the escape probability for resonance radiation ϵ for an electron temperature $T_e = 3000$°K (0.25 ev). A representative range for ϵ is between 0.01 and 0.001 for plasmas having dimensions of the order of 1 cm (see Ref. 5).

The second essential difference is that, in the MHD generator plasma, the degree of ionization of the gas corresponds nearly to that for equilibrium at the local electron temperature, whereas in the glow discharge, diffusion of the charged species to the walls is offset by electron-impact ionization, to give a much more complex relationship.

It is implicit in this description of the plasma that such nonequilibrium ionization cannot exist in a steady, homogeneous, stagnant gas. This is because the energy that is transferred from electrons to gas atoms must somehow be disposed of, if the difference between electron and gas temperatures is to be maintained. In a practical magnetogasdynamic device, the difference can be maintained by conversion of the thermal energy of the gas to kinetic energy.

Two experimental techniques have been employed in studying nonequilibrium ionization. In one, used by Kerrebrock,[3] convection is relied upon to maintain the gas temperature nearly constant while the gas flows through a test region. In the second, initially used by Ben Daniel, Bishop, Westendorp, Goldman, and Hurwitz,[4] an electric field is suddenly applied to a stagnant gas, and the nonequilibrium ionization is studied in the short interim after the electron temperature has risen but before the gas temperature rises.

One contribution of Ref. 3 was to point out that the experimentally observed variation of the conductivity of a flowing argon-potassium plasma was approximately predicted by a simple theory, which assumed 1) that the electron temperature was given *locally* by the balance between gain in the electric field and collisional loss which has been found to occur in gas discharges, and 2) that the ionization was in equilibrium at the *local* electron temperature.

The combination of these two assumptions led to an expression for the conductivity of the gas as a function of the current density, and hence to a nonlinear conduction law. Theory and experiment were brought into agreement by increasing the theoretical rate of energy transfer from electrons to atoms to about ten times that expected for monatomic gases. The comparison is reproduced in Fig. 1.

The conclusion from the theory was that it should be possible to maintain high conductivities in gases at low temperatures, and this was verified for temperatures as low as 1500°K.

Ben Daniel and Tamor[5] investigated the validity of the second assumption by analysis of a three-level model for the ionizable atom. They found that radiation emitted in the transition from the first excited state to the ground state (resonance radiation) could lead to deviations from Saha equilibrium by depopulating the first excited state. However, they expressed their results in terms of a resonance escape probability ϵ and correctly concluded that, even for laboratory-sized apparatus (1 cm), the Saha equation should be valid for seed concentrations greater than 0.3×10^{16} cm^{-3} (0.1% at 1 atm and 2000°K). Their plot of N_e/N vs N and ϵ is reproduced in Fig. 2 for an electron temperature of 0.25 ev.

Ben Daniel and Bishop[6] measured the conductivity of a cesium-helium mixture in a pulse-type experiment. The results were very similar to those obtained in the experiment of Ref. 3. Since the gas temperature was only about 250°C in their experiment, their results strongly supported the conclusion that high conductivities could be maintained at low gas temperatures. For a high cesium concentration (0.32×10^{15} cm^{-3}), the measured electric field was about twice that predicted by the simple theory. The effect of radiation on the ionization equilibrium was advanced as an explanation for the discrepancy between theory and experiment, but this is not a satisfactory explanation since rather large resonance escape probabilities must be invoked, whereas, as Ben Daniel and Tamor[5] pointed out, the actual resonance escape probability is very small.

Robben[7] measured the conductivity of an argon-potassium plasma produced by a large plasmajet, with results very similar to those of Ref. 3, although the conductivities were somewhat higher. He also attempted to operate a generator with nonequilibrium ionization, and was partially successful, as will be noted in Part II. His results are shown in Fig. 3.

Since this work, a considerable amount of both theoretical and experimental work has been done, with the result that there is now convincing evidence: first that Saha equilibrium at the electron temperature is attained in plasmas having sufficiently high electron densities to be useful in MHD generators, and second that the most important correction to the simple theory is for the loss of energy from the electron gas by resonance radiation. We shall consider these two points in turn.

Validity of the Saha Equation

As was noted previously, the theory of Ben Daniel and Tamor[5] predicted that the Saha equation should be valid for practical purposes if the escape probability for resonance radiation was less than about 0.01. It appears from the work of Byron, Bortz, and Russell[8] that the electron cross section for excitation to the first excited state of an alkali-metal atom is about 100 times as large as that assumed by Ben Daniel and Tamor. Their conclusions are therefore very conservative. The effect of the resonance radiation on the population of the first excited state was estimated in Ref. 9 by taking the (empirical) Joule heating rate as an upper limit to the radiant-energy loss and using the excitation cross sections of Byron et al. An error was made in Ref. 9 in the estimation of the cross section, but, at an electron density of 10^{13} cm^{-3}, the rate of excitation by electron collisions should in fact exceed the rate of de-excitation by radiation by a factor of 10^4.

Thus, the theoretical studies have uniformly concluded that the degree of ionization of the alkali-metal seed should be given quite accurately by the Saha equation evaluated at the electron temperature, if the density is high enough.

All of these conclusions, however, are predicated on the assumption that the free electrons have a Maxwellian energy distribution. As noted in Refs. 5 and 9, this is not likely to be the case below an electron mole fraction of about 10^{-6}. Below this value, the electrons are influenced more by energy exchange with atoms than by mutual energy exchange.

Perhaps the most direct experimental evidence for the validity of the Saha equation is offered by the measurements of Sheindlin, Batenin, and Asinovsky.[10] In their experiment, carried out in a flowing argon-potassium plasma produced by an arc, the electron temperature was measured directly by observation of the intensity of the $4s^2S_{1/2} - 5p^2P_{1/2}$ transitions in potassium. The conductivity computed from the *measured* electron temperature was then compared to the measured conductivity. The agreement is excellent at electron temperatures above 3000°K.

This conclusion is supported by the work of Zukoski, Cool, and Gibson,[11] who compare the *variation* of the electron temperature, inferred from the variation of the intensity of the resonance lines, with the variation of electron temperature computed from measured conductivities. Again the agreement is excellent for electron temperatures above 3000°K.

We have no such direct experimental evidence for lower electron temperatures, but from the conductivity data to be presented later, we can infer that the Saha equation is valid for electron densities as low as 10^{13} cm^{-3}.

Effect of Radiation on the Energy Balance

It is clear now that the energy loss from the electron gas by resonance (and other line) radiation is the most important correction to be made to the simple theory at high electron densities. Of course the direct loss from the electron gas is by inelastic collisions, but because nearly exact detailed balancing of the excitation and de-excitation rates exists, every photo-de-excitation in which the photon escapes the gas implies an excitation by a free electron. Thus, the energy of the emergent photon is taken indirectly from the free electron gas.

The magnitude of the radiant-energy loss has been computed in detail by Lutz[12] for an argon-cesium plasma. His major conclusions are that the principal part of the radiant-energy loss is due to the resonance lines for electron temperatures below 3000°K, and that, although these lines are strongly self-absorbed, the energy loss can be large because pressure broadening greatly increases their width. The magnitude of the computed energy loss is approximately that required to explain the observed conductivities.

This conclusion has been verified by Zukoski, Cool, and Gibson,[13] who have compared the two-temperature model, corrected for radiant energy loss, with their data obtained by a technique using a pulsed electric field in a flowing argon-potassium plasma. For the range of current densities (electron temperatures from 2500° to 3500°K) covered by their measurements, the agreement is excellent.

Revised Theory

We are therefore in a position to suggest a revised theory, accounting for radiation, which will predict the behavior of the conductivity rather accurately in the range of conductivities of interest. The electron energy balance takes the form

$$\frac{j^2}{\sigma} = \mathbf{J}\cdot\mathbf{E}' = \left[\sum_i \delta_i \frac{m_e}{m_i}\nu_{ci}\right] n_e\tfrac{3}{2}k(T_e - T_a) + Q_r \quad (1)$$

where $\mathbf{J}$ is the current density, $\mathbf{E}'$ is the electric field measured in the fluid, m_e and m_i are the masses of the electron and ith atomic species, ν_{ci} is the collision frequency with the ith species, and T_e and T_a are the electronic and gas temperatures; δ_i is approximately 2 for atomic species but may be large for molecular species; Q_r represents the radiant energy loss and is a function of the gas composition and density, of

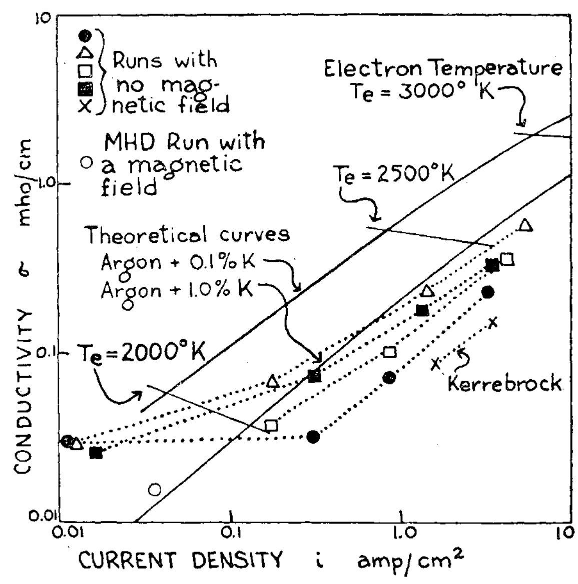

Fig. 3 Comparison of two-temperature conduction theory with measurements taken in flowing potassium-seeded argon. The best agreement between theory and experiment is again obtained for $\delta \approx 20$. One point exhibiting slight "magnetically induced nonequilibrium ionization" is shown. This is the first reported evidence for electron heating in an MHD generator (see Ref. 7).

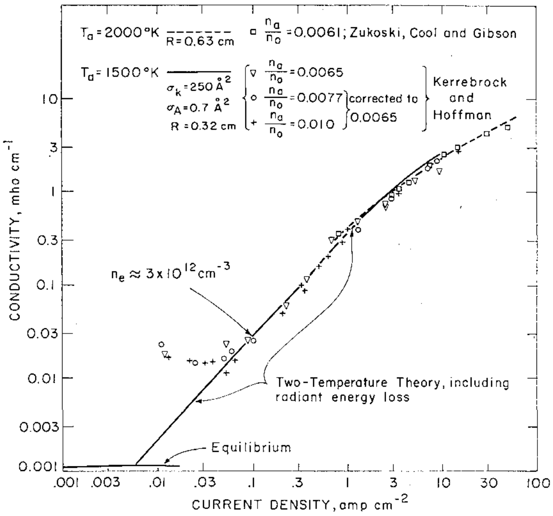

Fig. 4 Comparison of the two-temperature theory, revised to account for energy loss from the electrons by resonance radiation, with data of Kerrebrock and Hoffman,[16] and Zukoski, Cool, and Gibson.[13] The agreement is within the accuracy of the cross sections for current densities larger than about 0.1 amp-cm^{-2}, which corresponds to an electron concentration of about 3×10^{12} cm^{-3}. Below this value, the conductivity is higher than that predicted by theory.

the electron temperature, and of the geometry. It is not in general a local property but must be derived by the solution of a very complicated transport equation in which the local Joule heating enters as a source term. The approach followed by Lutz[12] and by Zukoski, Cool, and Gibson[13] was to compute the energy flux from a uniform slab of gas and divide by the volume of the slab to obtain a volumetric energy loss. It is difficult to evaluate the validity of this approximation at present. We can only say that it must break down for plasmas of very large dimensions, in which the boundaries will be more strongly cooled than the center. A detailed calculation of the transport of resonance radiation has been carried out by Holstein[14] for a decaying plasma. In the absence of a complete solution for the present (steady-state) problem, we shall use his escape factors and assume the loss to be uniformly distributed.

The radiation loss is then given by

$$Q_r = \sum_j G_j h\nu_j n_j A_{j\rightarrow 0} \tag{2}$$

where $h\nu_j$ is the energy of the transition from the jth level to the ground state, n_j is the density of the jth state, and $A_{j\rightarrow 0}$ is the Einstein spontaneous emission probability.

The resonance escape factor G_j depends only on the gas composition and on the geometry. For the case of dispersion broadening, Holstein gives the formulas

$$G_j = 1.150 \left(\frac{2g_0\gamma_p\nu_j^2}{c^2 n_0 g_j \gamma d}\right)^{1/2}$$
$$G_j = 1.115 \left(\frac{2g_0\gamma_p\nu_j^2}{c^2 n_0 g_j \gamma R}\right)^{1/2} \tag{3}$$

for a plane slab of thickness d and a cylinder of radius R. Here g_0 and g_j are the statistical weights of the ground state and of the excited state, n_0 is the density of atoms in the ground state, and γ and γ_p are the natural and total line widths.

For the alkali metals, the first level is a doublet, and Eq. (2) becomes

$$Q_r = \frac{2\pi e^2 n_0 h}{m_e \lambda^3 \epsilon_0 g_0}\,[G_1 g_1 f_{1\rightarrow 0} + G_2 g_2 f_{2\rightarrow 0}]\,e^{-h\nu/kTe} \tag{4}$$

where λ is the wave length of the doublet, g_1, g_2, and g_0 are the statistical weights of the members of the doublet and the ground state, and the $f_{1\rightarrow 0}$ and $f_{2\rightarrow 0}$ are their emission oscillator strengths. Values of gf and λ for the alkali metals are collected in Table 1.[15]

The theory is completed by the addition of the Saha equation, which relates n_e to T_e and the density of seed atoms, plus an expression relating the conductivity σ to the electron concentration, electron temperature, and gas properties. From these expressions, σ can be determined as a function of the gas properties and of the current density.[3]

The prediction of the two-temperature theory, corrected for radiant-energy loss, is compared with some of the experimental results of Kerrebrock and Hoffman,[16] and of Zukoski, Cool, and Gibson[13] in Fig. 4. The solid theoretical curve was computed from the preceding formulas. The dashed curve was given by the latter authors.[13]

The agreement is quite satisfactory; over a range of three factors of 10, the deviations are no larger than the uncertainty in the cross section of potassium. But below a current density of 0.1 amp-cm^{-2} or an electron density of 3×10^{12} cm^{-3}, the conductivity seems to rise rather than fall to the equilibrium value. The reason for this is not clear at present, although the upturn does occur just where simple arguments indicate the electrons should become non-Maxwellian because electron-electron energy exchange becomes equal to electron-atom energy exchange.[5, 9]

For electron concentrations above approximately 3×10^{12} cm^{-3}, we seem justified in concluding that the two-temperature theory is completely satisfactory for atomic gases, if corrected for the *energy loss* due to resonance radiation.

Wet Alkali-Metal Vapors

There is some interest in pure alkali-metal vapors as working fluids for MHD generators, because of the advantages of the Rankine cycle over the Brayton cycle.

Rowe[17] has reported a detailed analytical study of the behavior of a wet potassium plasma, together with preliminary experimental results. They indicate that the principal difference between such a plasma and the noble gas plasma lies in the very effective energy exchange between electrons and droplets of alkali metal. His measurements indicated an increase of a factor of 50 in the electronic energy loss over that of a dry gas. It was inferred from this that the droplets were approximately 10^{-7} cm in diameter. Smith[18] has reported an analytical study of this type of plasma, the results of which appear to agree with the analytical results of Rowe.

II. Generator Characteristics

Let us now consider whether the plasma conditions described previously can be realized in an MHD generator. There are several questions to be resolved. First, we may ask whether the electric field can be made large enough to

Table 1 Oscillator strengths of resonance lines in the alkali metals

Atom	Wavelength, Å	$g_i f_{i\rightarrow 0}$
Li	6707.84	0.80
Na	5889.95	0.95
	5895.92	0.47
K	7664.91	1.4
	7698.98	0.70
Rb	7800.23	2.7
	7946.60	1.2
Cs	8521.10	1.4
	8943.50	0.57

elevate the electron temperature, given that the plasma is homogeneous and stable. Secondly, we may ask how long a flow length is required for the gas to relax to the nonequilibrium state. Finally, we may ask whether the nonequilibrium state is stable, once achieved. Any experimental study is likely to involve all of these questions simultaneously. They can be treated separately in theory, however, so we shall consider them as distinct questions.

Attainment of Electron Heating: Theoretical Arguments

The conditions under which electron heating should occur in an ideal MHD generator were set forth by Hurwitz, Sutton, and Tamor.[19] By equating the Joule heating rate to the electronic energy loss due to elastic collisions, they deduced that in a Faraday generator the electron temperature should satisfy the relationship

$$T_e/T_a = 1 + (2\gamma/3\delta)\beta^2 M^2(1 - K)^2 \qquad (5)$$

where γ is the ratio of specific heats, β is the Hall parameter, M is the Mach number of the flow in the duct, and K is the ratio of the load voltage of the generator to its open circuit voltage. In deducing this relation, it was assumed that the axial or Hall current was zero everywhere in the plasma. This form of the energy balance makes it quite clear that high Mach numbers and high Hall parameters are desirable for nonequilibrium generators.

Of course we know now that this expression should be corrected for radiant energy loss, but in generators having minimum cross-sectional dimensions greater than 10 cm, this correction should be small. The importance of radiation lies chiefly in the proper interpretation of small-scale laboratory experiments.

The roles of the Mach number and Hall parameter can be clarified by rewriting Eq. (5) in terms of the stagnation temperature T_s as[19]*

$$\frac{T_e}{T_s} = \frac{1 + H[(\gamma - 1)/2]M^2}{1 + [(\gamma - 1)/2]M^2} \qquad (6)$$

where

$$H = \frac{4}{3\delta}\left(\frac{\gamma}{\gamma - 1}\right)\beta^2(1 - K)^2 \qquad (7)$$

From this form of the expression, it is clear that the electron

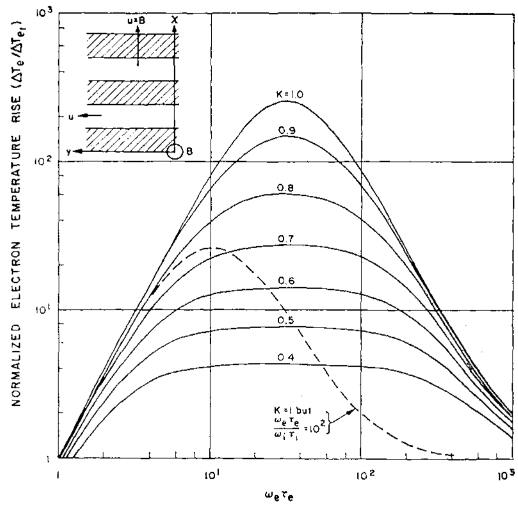

Fig. 6 The effect of nonuniformities on the electron temperature rise in an MHD generator. As in Fig. 5, K is the ratio of conductivities of alternate layers of gas, and the curves are drawn for $\omega_e\tau_e/\omega_e\tau_e = 10^3$, except for the dashed curve (see Ref. 20).

temperature can be larger than the stagnation temperature, provided $H > 1$. Since it is T_s that is limited by the heat source, we may say that the requirement for achieving magnetically induced nonequilibrium ionization is that $H > 1$. For a reasonable value of K, say $\frac{3}{4}$, β must be larger than 3. If $H > 1$, then a high Mach number is beneficial. The implications of this relation for Faraday and Hall generators are more fully developed in Ref. 19.

It has long been a familiar fact to gas dynamicists that high Mach number flows are subject to large losses of stagnation pressure, due to shocks. Similarly, flows with large Hall parameters are subject to large electrical losses due to Hall currents induced by nonuniformities. Karlowitz and Halasz refer to losses caused by the nonuniformity of the ionization produced by their electron beams. Rosa[20] has studied a model in which the fluid is stratified in layers normal to the current flow. He derived a "uniformity factor" G. The effective conductivity, Hall field, and electron temperature rise can all be expressed in terms of this factor. In particular, the Hall electric field becomes

$$E_x = -(\langle\Omega\rangle/G)\langle 1 - \eta\rangle uB \qquad (8)$$

where $\langle 1 - \eta\rangle uB$ is the average transverse electric field measured in the gas, and $\langle\Omega\rangle$ is the average value (in the direction of current flow) of

$$\Omega = \beta_e/(1 + \beta_e\beta_i) \qquad (9)$$

Here, β_e and β_i are the electron and ion Hall parameters. If we denote the transverse electric field measured in the gas as $E_y' = E_y - uB$, this expression becomes $E_x/E_y' = \langle\beta_e\rangle/G$ when ion slip can be neglected. The factor $1/G$ is shown as a function of $\beta_e = \omega_e\tau_e$ in Fig. 5, which is taken from Ref. 20. The parameter K is the ratio of the conductivities of the fluid in alternate layers of equal width.

From these results, Rosa has computed the ratio of the electron temperature rise to that in a uniform gas having $\beta_e = 1$. His results, which account also for ion slip, are reproduced in Fig. 6.

Kerrebrock[21] has generalized Eq. (6) to include such effects by writing H as

$$H = \frac{4}{3\delta}\left(\frac{\gamma}{\gamma - 1}\right)\beta^2\left(\frac{E_y'}{uB}\right)^2\frac{[1 + (E_x/E_y)^2]}{1 + \beta^2} \qquad (10)$$

For an ideal Faraday generator, $E_x = -\beta E_y'$; $E_y'/uB = K - 1$, and this expression reduces to Eq. (6). However, if

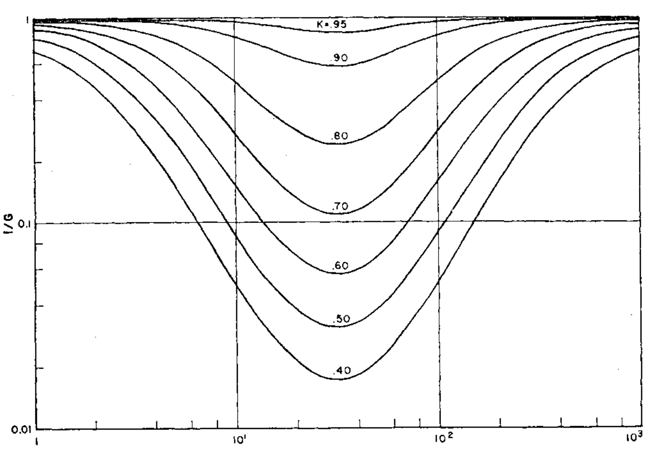

Fig. 5 The factor $1/G$ by which the maximum attainable Hall field is reduced by the presence of nonuniformities, as a function of electron Hall parameter $\omega_e\tau_e$. The ratio of conductivities of alternate layers of gas is K. The rise at large values of $\omega_e\tau_e$ is caused by ion slip. The curves are drawn for $\omega_e\tau_e/\omega_i\tau_i = 10^3$ (see Ref. 20).

* The authors of Ref. 19 did not introduce the parameter H, but their argument is equivalent to this one.

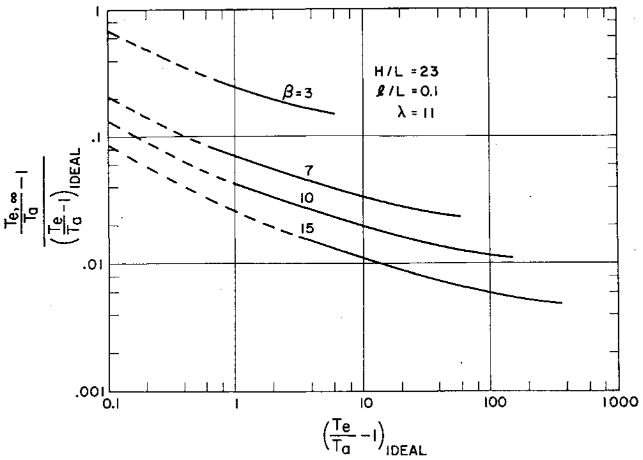

Fig. 7 Ratio of the actual electron temperature rise to ideal electron temperature rise in an MHD generator with segmented electrodes. The quantity H/L is the ratio of channel height to electrode spacing; l/L is the ratio of electrode width to electrode spacing; $\lambda = \epsilon/2kT_a$, where ϵ is the ionization potential and T_a the gas temperature; β is the Hall parameter and $(T_e/T_a - 1)_{ideal}$ is the ideal electron temperature rise (see Ref. 21).

E_x is much less than its ideal value, H is greatly reduced and the chance of observing electron heating is small. Furthermore, if E_y' is reduced by electrode voltage drops, H is also reduced. A similar relation has been obtained by Talaat.[22]

It has been suggested in Ref. 21 that segmented electrode generators designed to produce electron heating may be subject to severe losses of the type described by Rosa. This was argued qualitatively by noting that, at the surface of the insulating segments in the electrode wall, the normal current density $j_y = 0$. From a balance between elastic energy losses and Joule heating, it follows that at the surface of these insulators

$$T_e/T_a = 1 + (2\gamma/3\delta)\beta^4M^2(1 - K)^2 \tag{11}$$

for an ideal generator. Comparing this with Eq. (5), we see that $T_e - T_a$ should be β^2 times as large at the insulator as in the bulk of the gas. Thus, in a generator designed to produce electron heating, the heating should occur first near the electrodes. This would produce a stratification normal to the current flow. An approximate theory for generator performance, including this effect, has been given in Ref. 21.†

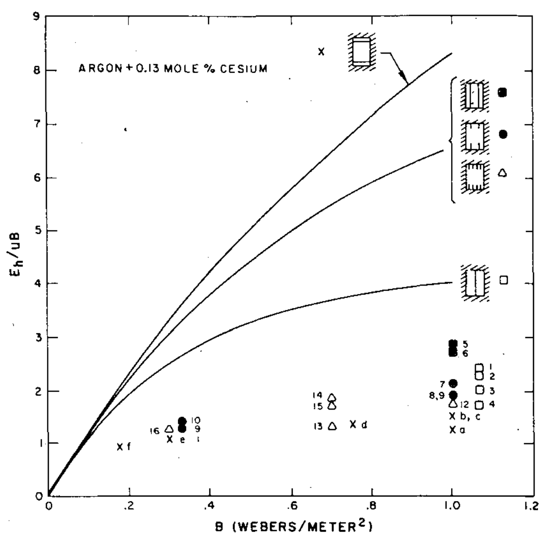

Fig. 8 Hall electric fields measured in segmented electrode generators, compared with the theoretical predictions of Hurwitz, Kilb and Sutton[27] (top curve) and Carlson[28] (lower two curves). The small diagrams indicate the electrode geometry (see Ref. 23).

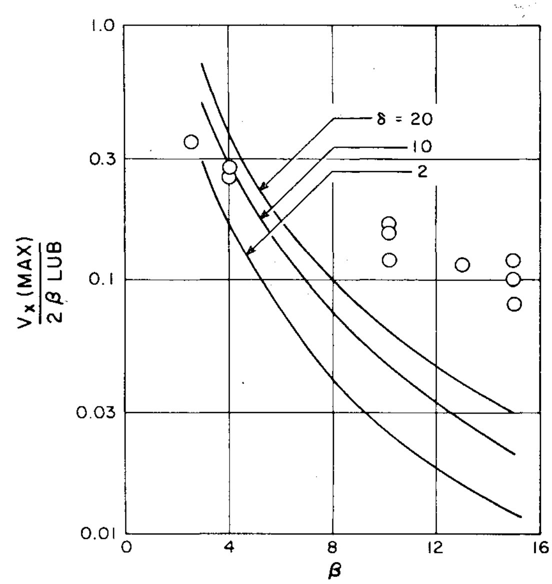

Fig. 9 The ratio of actual Hall voltage to ideal Hall voltage as a function of Hall parameter β, and the effective energy loss factor δ, compared to data of Klepeis and Rosa,[25] for $H/L = 23$ (see Fig. 7). The data presented here correspond to the triangles and crosses of Fig. 8 (see Ref. 21).

The conclusion reached from this analysis was that the maximum attainable Hall voltage and, therefore, the maximum attainable electron heating can be greatly reduced in a segmented electrode generator. The predicted electron temperature rise is given as a function of the ideal temperature rise in Fig. 7. The quantity H/L is the ratio of channel height to electrode spacing, whereas $(T_e/T_a - 1)_{ideal}$ is the value given by Eq. (5).

Attainment of Electron Heating: Experiments

Most of the attempts to produce nonequilibrium ionization by electron heating in MHD generators have been made with channels employing segmented electrodes. One experiment has been conducted with a "disk" type of Hall generator in which the conduction currents are closed internally. The characteristics of these experiments are summarized in Table 2. The data given include the type of plasma source, the characteristics of the plasma, the channel geometry, the equilibrium conductivity at the given gas conditions, and the actual conductivity. Finally, the magnitude of the parameter H, defined by Eq. (10), is given where enough data are available to allow it to be estimated.

The quantities with superscript c have been estimated from data given in the reference. They are not values given by the investigators themselves, and the present author therefore must assume responsibility for their correctness.

It should be noted first that, from the simple result [Eq. (11)], the electron temperatures in all of these experiments should have been sufficiently high to produce conductivities in the range from 1 to 10 mho-cm^{-1}, depending on the exact values of β_∞, M_∞, and K.

The first reported experimental evidence for nonequilibrium ionization in an MHD generator is that of Robben,[7] who unfortunately obtained only one data point, which is given in Fig. 3. It was obtained with a rather large generator, powered by an a.c. arc. Evidently for this reason, the experiment was very noisy, and, in addition, trouble was ex-

† Note in proof: For additional information on this point see Ref. 40.

perienced in obtaining uniform seeding with potassium. Nevertheless, a conductivity more than ten times the equilibrium value was observed. In the light of more recent experiments, and of the theory of Ref. 21, it seems possible that the performance of this generator was low because of Hall current shorting along the electrodes. Large electrode drops, such as this theory predicts, were in fact reported by Robben.[7]

Approximately two years later, experiments were reported by Zauderer,[23] by Talaat and Bienert,[24] and by Klepeis and Rosa.[25]

The experiment of Zauderer was carried out in xenon in a shock tube. High gas purity was obtained by thoroughly outgassing the tube. An essential difficulty encountered in this experiment was that the rate of ionization of the xenon, in the absence of impurities, was so slow that a steady state was not realized during the endurance of the experiment. In addition, the interaction of the flow with the magnetic field was so large that compression waves, traveling upstream from the shock, heated the gas above the temperature attained behind the shock. Nevertheless, measurements of conductivity at the end of the generator, taken just after the passage of the shock, indicated a sharp rise in conductivity with increasing magnetic field, and this has been interpreted by Zauderer as evidence of magnetically induced nonequilibrium ionization. Because of the ionization delay, however, the conductivities attained were less than the equilibrium conductivity at the gas temperature.

Talaat and Bienert's[24] experiment was conducted in a closed loop facility with an argon-cesium mixture of high purity. The gas temperatures were very low because of the failure of one of the metallic resistance elements used for heating the gas. Conductivities were measured which were somewhat higher than those for equilibrium at the gas temperature, but still far too low to be useful in an MHD generator. Talaat has presented an analysis of this experiment,[26] in which he has estimated the magnitude of E_x/E_y' [see Eq. (10)]. From his value, H has been estimated and is given in Table 2. This value is too small to explain the ratio between the reported experimental conductivity and the equilibrium value. However, it should be clear from the discussion of Part I that we should not expect simple arguments based on Saha equilibrium to be applicable at these very low electron densities.

Klepeis and Rosa[25] have reported two series of experiments carried out in argon-cesium mixtures produced by a graphite heater. In the first series of experiments, a segmented channel was used, and systematic measurements of the over-all Hall voltage were made, with the generator shorted in direction transverse to the flow. The Hall voltage was systematically much less than the ideal value. From the measured Hall fields, the value of H has been computed for a Hall parameter of 15 and is shown in Table 2. It is roughly correct to explain the slight increases in electron density (about 50%) observed in the experiment. The authors' comparison of their measured Hall fields with the predictions of the theory of Hurwitz, Kilb, and Sutton,[27] and Carlson[28] is reproduced in Fig. 8. The voltage is much less than that predicted by the theories. On the other hand, the theory of Ref. 21 predicts somewhat lower performance than was observed. This comparison is reproduced in Fig. 9. It was concluded that the electrode loss mechanism advanced in Ref. 21 was at least a possible explanation for the poor performance of the segmented generator.

Accordingly, Klepeis and Rosa have developed a disk-type Hall generator in which the flow is radial, the magnetic field is axial, and the transverse currents are shorted around the circumference of the disk. Thus, the electrodes and their attendant losses are eliminated. Magnetically induced nonequilibrium ionization has been conclusively demonstrated in this generator.[25] The voltage-current characteristic of the disk generator is reproduced in Fig. 10. It is distinctly nonlinear, indicating that the conductivity is a function of the current density. Klepeis and Rosa also show that the nonlinear characteristic appears only if the stagnation temperature (initial electron density) is sufficiently large that the ratio of the Joule heating rate to the radiant energy loss computed by Lutz[12] is equal to or greater than unity. This electron density is approximately that for which the data of Kerrebrock and Hoffman (Fig. 4) indicates a minimum in the conductivity.

From the data given in Ref. 23 for a β of 10, the actual Hall voltage in the disk generator was about one-half the theoretical value, which gives $H \approx 8$, and, according to Eq. (6), this should have led to an electron temperature of approximately 3800°K. Such a high value probably was not attained, but this may have been because the generator was too short, as we shall see in the next section.

Very recently, additional data has been reported for segmented electrode generators. Croitoru, Bekiarian, Graziotti, and Pithon[29] have operated a generator powered by an arc, with potassium-seeded argon as the working fluid. The Hall parameter was about 20. This generator, like that of Klepeis and Rosa, failed to produce more than a small fraction of its expected Hall voltage. From the small voltage

Table 2 Characteristics of nonequilibrium MHD generator experiments

Investigators → Plasma source →	Robben[7] a.c. arc	Zauderer[23] Shock tube	Talaat and Bienert[24] Metal heater	Klepeis and Rosa[25] Graphite heater	Klepeis and Rosa[25] Graphite heater	Croitoru et al.[29] d.c. arc
Generator channel geometry	Segmented	Segmented	Segmented	Segmented	Disk	Segmented
H, cm	5	5	5.1	7.6	20[a]	1.2
b, cm	2.5	5	0.64	1.3	0.4[b]	0.7
L, cm	1	1.3	1.7	0.32	2.5	0.4
Plasma characteristics						
Gas	A + 0.1% K	Xe	A + 0.1% Cs	A + 0.1% Cs	A + 0.1% Cs	A + 0.1% K
T, °K	1500	6400	863	1750	1850	2250
p, atm	1	1.1[c]	1	1	1	1
B, kgauss	13	15–30	11	10	6.3	16.3
ω/ν_c	9	10–20	3.8	15	9	20
M	1	...	0.56	0.5	0.5	0.57
Conductivity, mho-cm^{-1}						
Equilibrium	0.001	7	1.5×10^{-8}	0.03[c]	0.06[c]	0.31[c]
Measured	0.02	0.4–3	1.2×10^{-5}	0.04[c]	0.2[c]	1.0[c]
Hall field parameter, H	...	...	0.4[c]	1.6[c]	8[c]	0.3[c]

[a] Mean circumference ≈20 cm.
[b] Disk height ≈0.4 cm.
[c] Quantities so denoted have been estimated from data given in the reference.

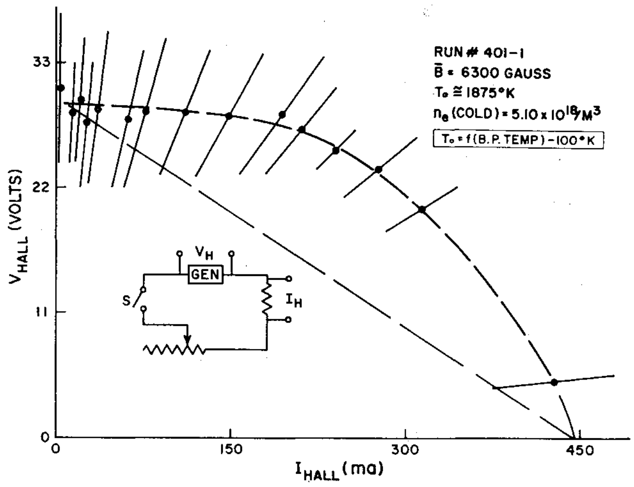

Fig. 10 The voltage vs current characteristic for a disk-type Hall generator. The radial current is I_{Hall}, and V_{Hall} is the radial voltage. The normal currents close azimuthally. The presence of nonequilibrium ionization is indicated by the curved characteristic, which indicates that the conductivity depends upon the current density. The bars on the data points indicate the magnitude of oscillations in the Hall voltage (see Ref. 23).

that was produced, H has been estimated and is given in Table 2. (For a 22-ohm load, see Ref. 29.) This small value of H is sufficient to explain the small but definite curvature of the voltage-current curves reported by Croitoru et al. which is a clear indication of nonequilibrium ionization.

Finally, Brederlow, Eustis, and Riedmüller[30] and Chapman[31] have reported much less than the expected performance from segmented electrode generators with plasma conditions suitable for the production of electron heating.

In summary, then, the disk generator experiment of Klepeis and Rosa has shown that magnetically induced nonequilibrium ionization can be produced in MHD generators, provided a sufficient Hall field recovery can be achieved, i.e., provided H can be made large enough. The segmented channel experiments of Croitoru et al. appear to confirm this, although not so clearly, because the increase in conductivity was smaller.

All of the segmented electrode experiments indicate a difficulty in recovering the Hall potential which is perhaps too universal to be due to poor insulators. An explanation of this phenomenon has been suggested in Ref. 21, but further investigation is clearly needed.

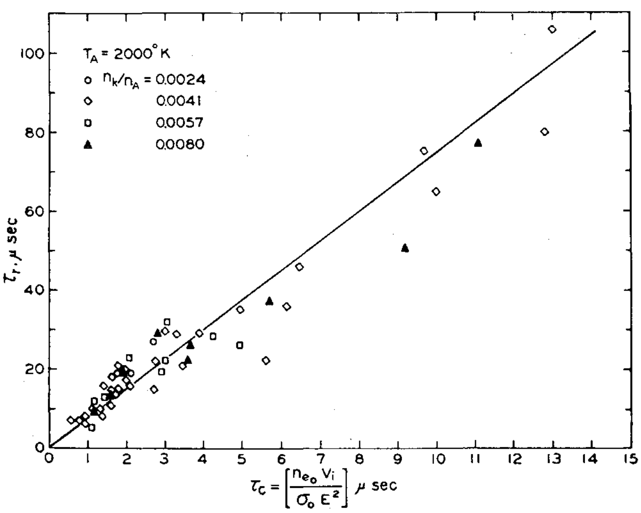

Fig. 11 Relaxation times τ_r measured by a transient technique in flowing potassium-seeded argon. The parameter τ_c measures the time required for the electric field E to supply the required ionization energy to the electrons (see Ref. 13).

Inlet Relaxation

At the inlet to a nonequilibrium MHD generator, the electron temperature and density must increase from their equilibrium (or perhaps stagnation state) values to the level dictated by the electric fields in the generator. This process of relaxation has been treated theoretically by Smith[32] and by Kerrebrock.[9]

Smith assumed that the electron temperature rises very rapidly to its asymptotic value, and that the rate of rise of the electron density is then determined by the ionization rate. In his analysis he neglected the fact that the electron temperature may be depressed by inelastic collisions as the degree of ionization is increasing. Such a depression would occur if the ionization rate were very fast.

In fact, in the limit of very fast ionization, the electron temperature and density should be related by Saha equilibrium during the relaxation process. This is the limit that was assumed by Kerrebrock.[9] In this case, the rate of relaxation is governed by the rate of energy input to the electrons. It is then found that the relaxation length is approximately

$$L = \frac{(3\gamma)^{1/2}}{4\delta}\left(\frac{m_a}{m_e}\right)^{1/2}\left(\frac{2\epsilon_i}{3kT_a}\right)^2\frac{M}{(T_e/T_a)_\infty}\,l \tag{12}$$

where $(T_e/T_a)_\infty$ is the asymptotic, electron-to-gas temperature ratio and l is the electronic-mean-free path.

From the analysis of measurements carried out in pulsed experiments, both Ben Daniel[33] and Zukoski, Cool, and Gibson[13] have concluded that ionization occurs by a multistep process. It is so rapid that it appears that the rate of relaxation in the inlet section is in fact governed by the rate of energy input to the electrons, as assumed in Ref. 9, rather than by the ionization rate itself. Zukoski, Cool, and Gibson have, on this basis, correlated the relaxation times measured in their experiments in terms of a characteristic time $\tau_c = n_{e0}\epsilon_i/\sigma_0 E^2$, where n_{e0} is the initial electron density, ϵ_i is the ionization energy, σ_0 is the initial conductivity, and E is the (constant) electric field applied to the plasma. They find that the actual time τ_r required for the conductivity to relax to 0.63 of its final value is about eight times τ_c.

From Eq. (12), this time τ_c is related to the true relaxation time L/U (or τ_r) by

$$\tau_r = \frac{L}{u} = \tau_c\left(\frac{\epsilon_i}{2kT_a}\right)\left[\left(\frac{T_e}{T_a}\right)_\infty - 1\right] \tag{13}$$

For values of $(T_e/T_a)_\infty$ greater than 1.3, this expression agrees very well with the data of Ref. 13 which are reproduced in Fig. 11. We therefore conclude that, for this range of conditions which encompasses the conditions of chief interest in MHD generators, the rate of relaxation is governed by the rate of energy input to the electrons, *not* by the rate of ionization.

For small values of $(T_e/T_a)_\infty - 1$, Eq. (13) predicts too small a value of τ_r, so that the energy input is not the rate-controlling factor. In the limit of very small $(T_e/T_a)_\infty - 1$, the ionization rate is probably controlling, as assumed by Smith.[32]

For the disk generator of Klepeis and Rosa, if $(T_e/T_a)_\infty \approx 1.5$ and $\delta \approx 10$ (which includes the radiation loss), then from Eq. (12) $L \approx 4$ cm, whereas the channel length is nearer 2 cm. Thus, in this case, only about one-half the asymptotic electron temperature rise should have been realized. This seems consistent with the observations.

We conclude, in agreement with Zukoski, Cool, and Gibson,[13] that the inlet relaxation is not likely to limit the performance of full-scale generators. However, it can be very important in the interpretation of small-scale experiments.

Stability

A rather detailed analytical treatment of the stability of the nonequilibrium plasma has been given in Ref. 9, where two types of instability are considered. The first, termed a static instability, is an instability to current concentration. The second is a kind of wave instability which, like the "Velikhov" instability,[34, 35] should occur only in plasmas with high Hall coefficients. There is a small amount of experimental evidence for the existence of each of these kinds of instability, as will be noted in the discussion of each.

Static Instability

The conclusion that can be drawn from the argument presented in Ref. 9 is that the seeded plasma becomes unstable if the seed is almost completely ionized and if, at the same time, Coulomb collisions dominate neutral collisions. This is because the electron temperature rises very rapidly in response to an increased current density, whereas the Coulomb cross section decreases. It can be seen, from the simple electron energy balance,

$$\frac{T_e}{T_a} = 1 + \frac{2m_a}{3\delta kT_a}\frac{j^2}{e^2n_e^2} \tag{14}$$

that such an instability should occur when j is such that $(T_e/T_a) - 1 > 1$ for $n_e \approx n_a$, the seed concentration. Thus the current density at which the plasma becomes unstable to current concentration should be proportional to n_a.

To the best of the author's knowledge, the only systematic data relevant to this question is that of Ben Daniel and Bishop,[4] who reported "constriction" in their pulsed experiment above certain limiting current densities. In Table 3, the current density for constriction is tabulated with the cesium concentration. Their ratio is nearly constant; furthermore the magnitudes are such that T_e may be quite large. Although these data are not conclusive, they certainly suggest that current constriction may occur in MHD generators wherever the current density is high, as for example near electrodes.

Wave Instability

A second kind of instability has also been considered in Ref. 9. There, it is proposed that in a nonequilibrium plasma an oscillation of the electron temperature and concentration may occur as a result of the combined effects of perturbations of electron concentration gradient and of electric field. The calculation assumes that the background gas is homogeneous.

It is concluded that such disturbances will grow in time intervals of the order of 10^{-5} sec, if their direction of propagation is nearly that of the steady current vector and if the Hall parameter is large. There is a sharp asymmetry about the current vector, with amplification on one side and attenuation on the other. The speed of propagation of the disturbance is predicted to be proportional to $T_e - T_a$, and to be much less than sonic velocity in the gas, unless $(T_e/T_a) - 1 \gg 1$.

There is clearly a strong connection between this type of disturbance and that considered by Velikhov[34] and McCune.[35] In the Velikhov instability, an acoustic oscillation is amplified by the body force, which results from the Hall currents induced by the density (Hall parameter) and conductivity fluctuation of the wave. In the "electrothermal wave," the oscillation is driven by the dissipation resulting from similar Hall currents.

It may be that an instability of either the Velikhov or "electrothermal" type (or more likely a combination of the two) has in fact arisen in nonequilibrium experiments with strong magnetic fields. Klepeis and Rosa[25] report violent fluctuations of the Hall voltage in their disk generator. In addition, violent fluctuations of the Hall potential have been noted in measurements conducted with the apparatus described in Ref. 16, when a magnetic field is applied perpendicular to the flow direction. It should be noted that, in both of these experiments, the plasma source is thermal, so that the possibility of inherent unsteadiness in the plasma is rather remote.

Much more theoretical and experimental work must be done before these phenomena can be fully understood or dismissed as unimportant. They are likely to be more important in experiments where the generator length is an interaction length or larger.

Other Methods for Producing Nonequilibrium Ionization

The data of Part I show that the degree of ionization of the high pressure *seeded* noble gas plasma is controlled by the rate of energy input to the electrons. Furthermore, they show that the electron gas, which can be taken to include the valence electrons of the seed species, tends very strongly to take on a Maxwell-Boltzmann distribution. This is because energy is transferred very readily between free electrons and also between free electrons and valence electrons. It follows that the physical properties of the plasma will not be sensitive to the way in which energy is supplied to this electron gas. This will be true provided that the electron density is larger than about 10^{13} cm^{-3}, which is certainly the range of chief interest for MHD generators.

Having established this point, we can see that the utility of any method for producing nonequilibrium ionization in a seeded gas is measured principally by its efficiency in converting energy from its source to electronic thermal or ionization energy. Since the direct Joule heating converts electrical output energy of the generator directly to electronic energy with an efficiency of unity, no scheme that uses electric power can be better than direct Joule heating from the standpoint of efficiency.

Such schemes may have other advantages, such as the capability for distributing the electron heating in the most advantageous way. Nevertheless, we must bear in mind that, in a heat engine, the basic thermodynamic efficiency of the components is of great importance, particularly if, as is the case here, they handle powers that are a large fraction of the output power of the system.

Electron beam ionization and photoionization

The technique of electron beam ionization was used by Karlowitz and Halasz[2] in their early experiments. They found that the resultant ionization was very nonuniform and that the recombination rate was much higher than they had expected.

Maitland[36] and Voshall and Emmerich[37] estimated the power required for photoionization by equating the rate of production to a recombination rate of the form αn_e^2. Their results seem overly optimistic since, as both Ben Daniel and Tamor[5] and Byron, Bortz, and Russell[8] have noted, at the high electron densities required in MHD generators, the

Table 3 Current density for constriction as a function of cesium concentration

n_{Cs}, cm^{-3}	j, amp-cm^{-2}	$j/n_{Cs} \times 10^{13}$
4.5×10^{13}	2.82	0.63
7.2×10^{13}	3.55	0.49
1.3×10^{14}	6.17	0.48
1.95×10^{14}	10	0.51
3.2×10^{14}	15	0.47
4.5×10^{14}	20	0.44

dominant recombination process involves an electron as the third body. The recombination rate is then of the form αn_e^3. Byron, Bortz, and Russell also noted that, since the recombination may add energy to a free electron, it may raise the electron temperature and so reduce the recombination rate somewhat. Under these conditions, the photoionization is merely another way to heat the electrons. It is subject to the efficiency criterion noted previously, and since the efficiency of light sources is generally low,[37] it does not appear promising.

A similar argument can be made for electron beam ionization. The recombination rate has been underestimated by recent investigators,[37] as well as by early ones[2]; but, since electron beams can be made quite efficient, this scheme cannot be eliminated by the efficiency criterion. It may, in fact, turn out that the capability for distributing the electron heating power in some particularly desirable way will make this scheme very attractive.

Delayed recombination

It has been proposed that delayed recombination be exploited to keep the conductivity of the gas high as it is expanded to supersonic velocities. Eschenroeder[38] has analyzed the recombination in air, assuming that the electron temperature is equal to the gas temperature. McNab and Lindley have studied the recombination in seeded noble gases.[39] The latter authors computed the elevation of the electron temperature, above the gas temperature, which should occur as a result of the higher specific heat of the electrons as compared to the atoms. But their calculation greatly underestimates the electron temperature rise because they did not include the ionization energy in their electronic energy balance. As noted by Byron, Bortz, and Russell,[8] the dominant recombination process will be one in which the ionization energy is given to an electron. This energy source is much more important than the initial thermal energy of the electrons.

We can estimate the electron temperature rise that will be produced by writing an electronic energy balance in the form[9]

$$-u(d/dx)[n_e(\tfrac{3}{2}kT_e + \epsilon_i)] = \delta(m_e/m_a)\nu_0\tfrac{3}{2}k(T_e - T_a)$$

where u is the flow velocity and ϵ_i is the ionization energy. Then, if we can allow n_e to decay to $1/e$ of its initial value in a generator length L, we find

$$\frac{T_e}{T_a} \approx 1 + \frac{2\gamma}{3\delta} M^2\beta\left(\frac{\epsilon_i}{euBL}\right)$$

Comparing this result with Eq. (5), we see that delayed recombination will be important if $\beta(1 - K)^2 < \epsilon_i/euBL$.

For cesium, with $u = 10^3 \text{msec}^{-1}$, $B = 10{,}000$ gauss, and $L = 1m$, $\epsilon_i/euBL = 3.9\times 10^{-3}$, whereas $\beta(1 - K)^2$ can be as large as unity for $K = \frac{3}{4}$. From this argument it appears that delayed recombination, as limited by energy transfer, will not be a competitive way of elevating the conductivity in full-scale generators, although it has been an important factor in laboratory experiments, where L has been, typically, a few centimeters.

Conclusions and Outlook

Two major positive conclusions seem to follow from the information that has been presented in this paper. These are as follows:

1) In the range of conditions suitable for MHD generators, the nonequilibrium conductivity of alkali-metal seeded noble gas plasmas is very well predicted by the simple two-temperature theory, which assumes Saha equilibrium, provided the radiant energy loss from the electron gas is properly accounted for.

2) Sufficient electron heating to produce conductivities as large as 1000 mho-m^{-1} can be produced in seeded noble gases at temperatures between 1500° and 2000°K, provided a large fraction of the theoretical Hall electric field can be realized at large Hall parameters.

On the negative side, it appears that it may be difficult to achieve satisfactory operation of segmented electrode generators at large Hall parameters in gases susceptible to electron heating. Furthermore, stability problems may arise in full-scale generators. Ionization by electron beams offers a possibility for avoiding some stability problems; however, it does not obviate the necessity for operation at large Hall parameters.

The principal task remaining is to develop an MHD generator that operates efficiently with atomic gases at large Hall parameters. If this can be done, the success of the nonequilibrium generator seems assured.

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